Punjab Technical University

Maximum Marks: 90 **Time: 90 Mins.**

Entrance Test for Enrollment in Ph.D Programme

Important Instructions

- Fill all the information in various columns, in Capital letters, with blue/black point pen for attempting the questions
- Use of calculators is not allowed.
- Make attempt by writing the answer in capital Letters in the box against each question number.
- All questions are compulsory. Each Question has only one right answer. No Negative marking for wrong answers.
- Questions attempted with two or more options/answers will not be evaluated.

Stream:	**......Applied Sciences ....**
Discipline	**......Mathematics ....**
Name	**...**
Fathers Name	**...**
Roll Number	**Date: 13-07-2014**
Signature of Candidate:	**...**
Signature of Invigilator	**...**

1. Let S_7 denote the symmetric group of all permutations of the symbols $\{1, 2, 3, 4, 5, 6, 7\}$. Pick out the true statements:
(i). S_7 has an element of order 10;
(ii). S_7 has an element of order 15;
(iii). the order of any element of S_7 is at most 12.

(a) (i) (b)(ii)

(c) (i) and (ii) (d) (ii) and (iii)

2. The number of non-zero elements in the field $\mathcal{Z}_p$, where p is an odd prime number, which are squares, i.e. of the form m^2, $m \in \mathcal{Z}_p$, $m \neq 0$:

(a) (p-1)/5 (b)(p-7)/2

(c) (p-1)/7 (d) (p-1)/2

3. Let $A \in \mathbb{M}_3(\mathbb{R})$ which is not a diagonal matrix. Pick up the cases when A is diagonalizable over $\mathbb{R}$:
(i). when $A^2 = A$;
(ii) when $(A - 3I)^2 = 0$;
(iii). when $A^2 + I = 0$.

(a) (i) (b)(ii)

(c) (iii) (d) none of these

4. Which of the following statements are true
(i). There exists an entire function $f : \mathbb{C} \to \mathbb{C}$ which takes only real values and is such that $f(0) = 0$ and $f(1) = 1$.
(ii). There exists an entire function $f : \mathbb{C} \to \mathbb{C}$ such that $f(n + \frac{1}{n}) = 0$ for all $n \in \mathbb{N}$
(iii). there exists an entire function $f : \mathbb{C} \to \mathbb{C}$ which is onto and which is such that $f(1/n) = 0$ for all $n \in \mathbb{N}$:

(a) (i) (b)(ii)

(c) (iii) (d) none of these

5. Let X and Y be metric spaces and let $f : X \to Y$ be a mapping. Then which of the following statements are true:
(i). If f is uniformly continuous, then the image of every Cauchy sequence in X is a Cauchy sequence in Y;
(ii). If X is complete and if f is continuous, then the image of every Cauchy sequence in X is a Cauchy sequence in Y;
(iii). If Y is complete and if f is continuous, then the image of every Cauchy sequence in X is a Cauchy sequence in Y;

(a) (i) and (ii) (b)(ii) and (iii)

(c) (i) only (d) (iii) only

6. Which of the following sets are connected?
(i). The sets $\{(x,\ y) \in \mathbb{R}^2 | xy = 1\}$ in $\mathbb{R}^2$.
(ii). The set of all symmetric matrices in $\mathbb{M}_n(\mathbb{R})$.
(iii). The set of all orthogonal matrices in $\mathbb{M}_n(\mathbb{R})$

(a) (i) (b)(ii)

(c) (iii) (d) none of these

7. Let $B \in \mathbb{M}_n(\mathbb{R})$ and let $b \in \mathbb{R}^n$ be a given fixed vector. Consider the iteration scheme

$$x_{n+1} = Bx_n + b,\ x_0 \text{ given.}$$

Pick out the true statements:
(i) the scheme is always convergent for any initial vector x_0.
If the scheme is always convergent for any initial vector x_0, then $I - B$ is invertible.
(iii). If the scheme is always convergent for any initial vector x_0, then every eigenvalue λ of B satisfies $|\lambda| < 1$.

(a) (i) and (ii) (b)(i) and (iii)

(c) (ii) and (iii) (d) (iii) only

8. Which of the following sets are countable?
(i). The set of all sequences of non-negative integers.
(ii). The set of all sequences of non-negative integers with only a finite number of non-zero terms.
(iii). The set of all roots of all monic polynomials in one variable with rational coefficients.

(a) (i) and (ii) (b)(ii) and (iii)

(c) (i) and (iii) (d) all the above

9. Five letters are addressed to five different persons and the corresponding envelopes are prepared. The letters are put into the envelopes at random. What is the probability that no letter is in its proper envelope?

(a) $\frac{11}{30}$ (b) $\frac{11}{50}$

(c) $\frac{5}{6}$ (d) no solution

10. What is the lengths of semi-axes of the ellipse whose equation is given by

$$5x^2 - 8xy + 5y^2 = 1.$$

(a) semi-major axis=1; semi-minor axis=1/3
(b) semi-major axis=5; semi-minor axis=5/6
(c) semi-major axis=2; semi-minor axis=2/5
(d) semi-major axis=3; semi-minor axis=3/5

11. Point of intersection of $x - y$ plane and the tangent line to the helix $x = cost,\ y = sint,\ z = t$ is:

(a) $x = cost + tsint,\ y = sint - tcost,\ z = 0$ (b) $x = cost - tsint,\ y = sint + tcost,\ z = 0$

(c) $x = tcost + sint,\ y = sint + tcost,\ z = tcost$ (d) $x = tcost - sint,\ y = 0,\ z = sint - tcost$

12. Intrinsic equations of the curve $x^2 + y^2 = a^2$ is:

(a) $\kappa = \frac{1}{a},\ \tau = 0$ (b) $\kappa = 0,\ \tau = \frac{1}{a}$

(c) $\kappa = a,\ \tau = \frac{1}{a}$ (d) $\kappa = \frac{1}{a},\ \tau = a$

The curve whose intrinsic equations are $\kappa = \frac{1}{\sqrt{2as}},\ \tau = 0$ is:

(a) $x = cos\phi - \phi sin\phi;\ y = \phi cos\phi + sin\phi$ (b) $x = cos\phi + \phi sin\phi;\ y = cos\phi - \phi sin\phi$

(c) $x = a(cos\phi + \phi sin\phi);\ y = a(sin\phi - \phi cos\phi)$ (d) $x = a(cos\phi - \phi sin\phi);\ y = a(sin\phi + \phi cos\phi)$

14. The surface $x = u,\ y = v,\ z = u^2 + v^3$ is elliptic if:

(a) $v < 0$ (b) $v > 0$

(c) $v = 0$ (d) $v \leq 0$

15. Curve joining the points $(0,\ 0)$ and $(a,\ b)$ such that the particle sliding along the curve takes minimum time is:

(a) Paraboloid (b) Hyperboloid

(c) Cycloid (d) None of these

16. The shortest distance between the points $(x_1,\ y_1)$ and $(x_2,\ y_2)$ is given by:

(a) $y = c_1x^2 + c_2$ (b) $y = c_1x^2$

(c) $y = c_1x^3 + c_2$ (d) $y = c_1x + c_2$

17. The extremal of $\int_0^1 yy'^3 dx$, y(0)=0, y(1)=1 is:

(a) $y = x^{4/3}$ (b) $y = x^{3/4}$

(c) $y = x^{2/3}$ (d) $y = x^{3/2}$

18. Extremal of the functional $\int_0^1 \frac{1}{2} y''^2 dx$ is:

(a) $y = x^2$ (b) $y = \frac{1}{3}x^2$

(c) $y = x^3$ (d) $y = \frac{1}{2}x^2$

19. Solution of the Fredholm equation $y(x) = e^x - x + \int_0^1 (1 - e^{xt})xy(t)dt$ is:

(a) $y(x) = 1$ (b) $y(x) = -1$

(c) $y(x) = \frac{1}{2}$ (d) $y(x) = -\frac{1}{2}$

20. Solution of the Volterra equation $\int_0^x e^{x-t} y(t)dt = x$ is:

(a) $x - 1$ (b) $x^2 - 1$

(c) $1 - x$ (d) $1 - x^2$

21. The Fredholm equation corresponding to the boundary value problem $y'' = f(x),\ y(0) = 0,\ y(1) = 0$ is:

(a) $\int_0^x (x-1)xf(t)dt + \int_x^1 (t-1)tf(t)dt$ (b) $\int_0^x (x-1)xf(t)dt + \int_x^1 (t-1)tf(t)dt$

(c) $\int_0^x (x-1)tf(t)dt - \int_x^1 (t-1)xf(t)dt$ (d) $\int_0^x (x-1)tf(t)dt + \int_x^1 (t-1)xf(t)dt$

22. The Volterra equations corresponding to initial value problem

$$y'' + y = 0,\ y(0) = 0,\ y'(0) = 1$$

is:

(a) $z(x) = x - \int_0^x (t+x)z(t)dt$ (b) $z(x) = -x + \int_0^x (t+x)z(t)dt$

(c) $z(x) = -x + \int_0^x (t-x)z(t)dt$ (d) $z(x) = -x - \int_0^x (t+x)z(t)dt$

23. Solution of Integral equation $y(x) = cotx + \lambda \int_{-\pi/4}^{\pi/4} tanty(t)dt$ is:

(a) $tanx - \lambda\pi/2$ (b) $cotx + \lambda\pi/2$

(c) $-tanx + \lambda\pi/2$ (d) $-cotx + \lambda\pi/2$

24. The eigen value of the integral equation $y(x) = \lambda \int_0^{2\pi} sinx\ cost\ y(t)\ dt$ is:

(a) 2 (b) -3

(c) 3/2 (d) None of these

25. The inverse Laplace transform of $\dfrac{p^2+1}{p(p+1)(p+2)}$.

(a) $\frac{1}{2}e^{-t/2} - sint$ (b) $\frac{1}{2}e^{t/2} + cost$

(c) $\frac{1}{2} - 2e^{-t} + \frac{5}{2}e^{-2t}$ (d) $-2 - \frac{t}{2} + e^{2t}$

26. Laplace transform of the function $f(t) = t$ for $0 \leq t < 1$ and $f(t+1) = f(t)$, for all t is:

(a) $\dfrac{1}{p^2} - \dfrac{1}{p(e^p - 1)}$ (b) $\dfrac{1}{p(1+e^{-ap})}$

(c) $\dfrac{1}{p^2}tanh(\dfrac{ap}{4})$ (d) $\dfrac{1}{p^2(1+e^{-p})}$

27. Laplace transform of $J_1(t)$, i.e. Bessel function of order one is:

(a) $1 - \dfrac{p}{\sqrt{p^2+1}}$ (b) $\sqrt{p^2+1} - p$

(c)$1 + \dfrac{p}{\sqrt{p^2-1}}$ (d) $\sqrt{p^2-1} + p$

28. Fourier Sine transform of xe^{-ax} is:

(a) $\dfrac{2s}{(a^2+s^2)^2}$ (b) $\sqrt{\dfrac{2}{\pi}}\dfrac{2s}{(a^2+s^2)^2}$

(c) $\sqrt{\dfrac{2}{\pi}}\dfrac{2as}{(a^2+s^2)^2}$ (d) $\dfrac{2as}{(a^2+s^2)^2}$

29. Fourier Cosine transform of xe^{-ax} is:

(a) $\sqrt{\dfrac{2}{\pi}}\dfrac{2s^2}{(a^2+s^2)^2}$ (b) $\sqrt{\dfrac{2}{\pi}}\dfrac{(a^2-s^2)}{(a^2+s^2)^2}$

(c) $\sqrt{\dfrac{2}{\pi}}\dfrac{2as^2}{(a^2+s^2)^2}$ (d) $\sqrt{\dfrac{2}{\pi}}\dfrac{s^2}{(a^2+s^2)^2}$

30. What is the dual of the following
Minimize $Z = 3x_1 + x_2$ with constraints
$x_1 + x_2 \geq 1;\ 2x_1 + 3x_2 \geq 2;\ x_1,\ x_2 \geq 0$:

(a) Maximize $-Z = 3x_1 + x_2$ with $x_1 + x_2 \leq 1;\ 2x_1 + 3x_2 \leq 2;\ x_1,\ x_2 \leq 0$
(b) Maximize $Z = -3x_1 - x_2$ with $x_1 + x_2 \leq 1;\ 2x_1 + 3x_2 \leq 2;\ x_1,\ x_2 \leq 0$
(c) Maximize $-Z = 3x_1 + x_2$ with $-x_1 - x_2 \leq 1;\ -2x_1 - 3x_2 \leq 2;\ x_1,\ x_2 \geq 0$
(d) Maximize $-Z = -3x_1 - x_2$ with $-x_1 - x_2 \leq -1;\ -2x_1 - 3x_2 \leq -2;\ x_1,\ x_2 \geq 0$

31. The singular solution of $p = log(px - y)$ is:

(a) $y = x(logx - 1)$ (b) $y = xlogx - 1$

(c) $y = logx - 1$ (d) $y = xlogx$

32. $e^{-x}(C_1 cos\sqrt{3x} + C_2 sin\sqrt{3x}) + C_3 e^{2x}$ is the general solution of :

(a) $\frac{d^3y}{dx^3} + 4y = 0$ (b) $\frac{d^3y}{dx^3} + 8y = 0$

(c) $\frac{d^3y}{dx^3} - 8y = 0$ (d) $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2 = 0$

33. Which of the following sets is connected:

(a) $\{z : |z| < 1\} \bigcup \{z : |z - 1| < 1\}$ (b) $\{z : -1 < |z| \leq 1\} \bigcup \{2\}$

(c) ϕ (d) $\{z : |z^2 - 1| < 2\}$

34. If the function $f : (X, \mathfrak{T}) \rightarrow (Y, \mathfrak{T})$ is homeomorphism, then the property of openness of f can be replaced by:

(a) f is continuous (b) f^{-1} is Continuous

(c) f is connected (d) f^{-1} is connected

35. If every subset of a topological space (X, T) is closed, then T is:

(a) Indiscrete Topology (b) Usual Topology

(c) Closed Topology (d) Discrete Topology

36. Let (X, d) be a metric space. Let $G \subseteq X$. Let $x \in G$ be any point such that $N_r(x) \bigcap G \neq \phi$ and $N_r(x) \bigcap G^c \neq \phi$. Then x is:

(a) Frontier point of G (b) limit point of G

(c) Boundary point of G (d) Interior point of G

37. The set of equations $x + y - 2z = 0$; $2x - 3y + z = 0$; $x - 5y + 4z = k$ is consistent for:

(a) $k = 0$ (b) $k = 1$

(c) $k = 2$ (d) $k = 5$

38. If the column vectors of a square matrix A are linearly dependent then:

(a) $|A| = 1$ (b) $|A| = \infty$

(c) $|A| \neq 0$ (d) $|A| = 0$

39. If A' is transpose of A, then:

(a) trace of $A' \neq$ trace of A (b) trace of $A' =$ trace of A

(c) trace of $A' <$ trace of A (d) trace of $A' >$ trace of A

40. Every diagonal element a_{ii} of a skew symmetric matrix is necessarily:

(a) Zero (b) Pure imaginary number

(c) a complex number (d) a real number

41. Rank of a singular matrix is:

(a) Equal to its order (b) Less than its order

(c) zero (d) Not defined

42. System of non-homogeneous linear equations is consistent if:

(a) Rank of a augment matrix is equal to Rank of the matrix
(b) Rank of a augment matrix is less than Rank of the matrix
(c) Rank of a augment matrix is always zero
(d) None of these

43. Vectors are Linearly dependent if the rank of their matrix is:

(a) Equal to the order of a matrix (b) Less than the order of a matrix

(c) zero (d) Not defined

44. Every infinite bounded set in $\Re^n$ possesses:
(a) Closed subset (b) Open subset

(c) Frontier point (d) Accumulation point

45. Let $\{Q_k\}$, $k = 1, 2, 3, ...$ be countable collection of nested non-empty sets in $\Re^n$ such that each set $\{Q_k\}$ is closed and $\{Q_1\}$ is bounded. Then $\bigcap_{k=1}^{\infty} Q_k$ is:

(a) open and non-empty (b) closed and empty

(c) open and empty (d) closed and non-empty

46. The series $\frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \, coskx + b_k \, sinkx)$ is the Fourier series of the function $f(x)$ if and only if

(a) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \, coskx \, dx, \, k = 1, \, 2, \, 3, \ldots$ and
$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \, sinkx \, dx, k = 0, \, 1, \, 2, \, 3, \ldots$

(b) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \, coskx \, dx, \, k = 0, \, 1, \, 2, \, 3, \ldots$ and
$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \, sinkx \, dx, k = 1, \, 2, \, 3, \ldots$

(c) $a_k = \frac{1}{\pi}\int_{0}^{\pi} f(x) \, coskx \, dx, \, k = 1, \, 2, \, 3, \ldots$ and
$b_k = \frac{1}{\pi}\int_{0}^{\pi} f(x) \, sinkx \, dx, k = 0, \, 1, \, 2, \, 3, \ldots$

(d) $a_k = \frac{1}{\pi}\int_{0}^{\pi} f(x) \, coskx \, dx, \, k = 0, \, 1, \, 2, \, 3, \ldots$ and
$b_k = \frac{1}{\pi}\int_{0}^{\pi} f(x) \, sinkx \, dx, k = 1, \, 2, \, 3, \ldots$

47. Random Variable is:

(a) Function from set of real numbers to sample space (b) Function from sample space to set of real numbers

(c) Function from sample space to sample space (d) real valued function on the set of real numbers

48. If there are n exhaustive number of events in an experiments, then probability of an event is defined if:

(a) Events are mutually exclusive (b) Events are equally likely

(c) Both mutually exclusive and equally likely (d) Events are independent

49. Poisson Distribution is a limiting case of Binomial distribution under the following conditions:

(a) $n \to 0; \; p \to 0$ (b) $n \to 0; \; p \to \infty$

(c) $n \to \infty; \; p \to \infty$ (d) $n \to \infty; \; p \to 0$

50. Sequence is:

(a) Function defined on set of real numbers

(b) Complex valued function defined on $\Re$
(c) Function defined on set of natural numbers
(d) function defined on set of integers

51. The problem $\frac{dy}{dx} = \frac{1}{x}$, $y(0) = 0$ has:

(a) infinite solutions (b) no solution

(c) unique solution (d) finitely many solutions

52. Solution of an equation $(yz + z^2)dx - xzdy + xydz = 0$ is:

(a) $y - cx = 0$ (b) $y + 1 = cx$

(c) $y = cx^2 + 1$ (d) none of the above

53. Which of the following is locally compact:

(a) Set of all rational numbers (b) Set of all real numbers

(c) Set of all irrational numbers (d) Set of all natural numbers

54. Let N and N' be normed linear spaces. Let $T : N \to N'$ be a linear transformation. Then T is continuous if and only if

(a) T is bounded (b) $T(S)$ is bounded in N' for S being closed sphere in N

(c) T is continuous at origin (d) All of the above

55. If $T : N \to N'$ is a linear transformation, then which of the following statement is true

(a) If T is continuous, then T is uniformly continuous
(b) T is continuous at any point of N, then it is continuous at every point of N
(c) T is bounded then T is uniformly continuous
(d) All of the above

56. A subset M of a metric space (X, d) is called nowhere dense if

(a) $(\bar{M})^o = \phi$ (b) $(M^c)^o = \phi$

(c) $(M^o)^o = \phi$ (d) $(\bar{M})^c = \phi$

57. Every complete metric space is a:

(a) Meagre (b) Non Meagre

(c) Dense (d) Nowhere dense

58. Which of the following is wave equation:

(a) $\nabla^2\phi = 0$ (b) $\nabla^2\phi = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$

(c) $\nabla^2\phi = \frac{1}{c}\frac{\partial\phi}{\partial t}$ (d) None of the above

59. Intervals on the real line are always:

(a) compact (b) seperable

(c) connected (d) no where dense

60. Limit of a sequence is:

(a) Always unique
(b) Unique only in Hausdorff spaces
(c) Unique only if the space is Lindeloff
(d) Uniqueness depends upon the point of convergence

61. What kind of singularity $tan\frac{1}{z}$ have at $z = 0$:

(a) Isolated essential singularity (b) Non isolated essential singularity

(c) Removable singularity (d) None of the above

62. If $f^{n+1}(x)$ is continuous at $x = a$ and $f^{n+1}(a) \neq 0$, then θ, which occurs in the Lagrange's form of the remainder $R_n = \frac{h^n}{n!}f^n(a + \theta h)$, as $h \to 0$ tends to the limit

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$

(c) $\frac{1}{n+1}$ (d) None of these

63. Generalized mean value theorem reduces to Lagrange's mean value theorem when:

(a) $g(x) = 1;\ h(x) = 1$ (b) $g(x) = 1;\ h(x) = x$

(c) $g(x) = x;\ h(x) = 1$ (d) $g(x) = x;\ h(x) = 0$

64. If f and F be both continuous on $[a,\ b]$ and are derivable on $(a,\ b)$ and if $f'(x) = F'(x)$ for all x in $(a,\ b)$ then $f(x)$ and $F(x)$ differ:

(a) by a constant in $[a,\ b]$ (b) by x in $[a,\ b]$

(c) by 1 in $[a,\ b]$ (d) never

65. On the curve $y = px^2 + qx + r$, $p \neq 0$, the chord joining the points for which $x = a$, $x = b$ is parallel to the tangent at the point given by:

(a) $x = \frac{1}{4}(a+b)$ (b) $x = \frac{1}{2}(a+b)$

(c) $x = \frac{2}{3}(a+b)$ (d) None of these

66. Let $f'(c) = f''(c) = ... = f^{n-1}(c) = 0$ and $f^n(c) \neq 0$. If n is even, then:

(a) $f(c)$ is not an extreme value
(b) $f(c)$ is a minimum value if $f^n(c) < 0$
(c) $f(c)$ is a maximum value if $f^n(c) > 0$
(d) $f(c)$ is a minimum value if $f^n(c) > 0$

67. The curved surface of right circular cylinder of greatest curved surface which can be inscribed in a sphere is:

(a) twice of the sphere (b) equal to the sphere

(c) one third of the sphere (d) one half of the sphere

68. For any matrix A:

(a) $\rho(A^tA) = \rho(A)$ (b) $\rho(A^tA) > \rho(A)$

(c) $\rho(A^tA) < \rho(A)$ (d) $\rho(A^tA) \leq \rho(A)$

69. If A is non-zero column matrix and B is non zero row matrix, then:

(a) $\rho(AB) = 0$ (b) $\rho(AB) = 1$

(c) $\rho(AB) = 2$ (d) $\rho(AB) = 3$

70. If x and y be two positive real numbers, then there exists a positive integer n such that $ny > x$. This property is referred to as:

(a) division algorithm (b) denseness property

(c)Euclidean algorithm (d) Archimedian property

71. A particular solution of n^{th} order differential equation contains:

(a) $n-1$ arbitrary constants (b) n arbitrary constants

(c) $n+1$ arbitrary constants (d) No constant

72. The family of curves represented by the differential equation $xdx - ydy = 0$ is of:

(a) circles (b) ellipses

(c) parabolas (d) hyperbolas

73. In a Poisson distribution if $2P(x = 1) = P(x = 2)$, then the variance is:

(a) 0 (b) -1

(c) 4 (d) 2

74. In a t distribution of sample size n, the degrees of freedom are:

(a) n-1 (b) n+1

(c) n (d) 1

75. A hypothesis is false but accepted, then there is an error of type:

(a) I (b) II

(c) III (d) no error

76. The order of convergence in Newton-Raphson method is:

(a) 2 (b) 3

(c) 0 (d) none

77. The value of $E^{-1}\nabla$ is:

(a) $\nabla - \nabla^2$ (b) $\nabla^2 - \nabla$

(c) $\dfrac{\nabla}{\nabla^2 - 1}$ (d) $\dfrac{\nabla^2 - 1}{\nabla}$

78. The order of the difference equation $y_{n+2} - 2y_{n+1} + y_n = 0$ is:

(a) 1 (b) 3

(c) 0 (d) 2

79. The solution of difference equation $y_{n+2} - 4y_{n+1} + 4y_n = 0$ is:

(a) $(c_1 + c_2 n)2^n$ (b) $2(c_1 + c_2 n)$

(c) $(c_1 + c_2 n + c_3 n^2)$ (d) $2^{n-1}(c_1 + c_2 n)$

80. Runge-Kutta method is used to solve:

(a) boundary value problem (b) initial value problem

(c) both (d) none

81. Which of the following equations is parabolic:

(a) $f_{xx} - f_x = 0$ (b) $f_{xx} + 2f_{xy} + f_{yy} = 0$

(c) $f_{xx} + 2f_{xy} + 4f_{yy} = 0$ (d) none

82. If $\mathbf{F} = ax\mathbf{I} + by\mathbf{J} + cz\mathbf{K}$, then $\int_s \mathbf{F} \cdot d\mathbf{S}$, S being the surface of a unit sphere, is:

(a) $(4/3)\pi(a+b+c)^2$ (b) 0

(c) $(4\pi/3)(a+b+c)$ (d) none of these

83. The directional derivative of $f(x,y) = (x^2 - y^2)/xy$ at $(1,1)$ is zero along a ray making an angle with the positive direction of $x-$axis:

(a) 45^o (b) 60^o

(c) 135^o (d) none of these

84. The work done by the force $\mathbf{F} = yz\mathbf{I} + zx\mathbf{J} + xy\mathbf{K}$ in moving a particle from the point (1, 1, 1) to the point (3, 3, 2) along the path c is:

(a) 17 (b) 10

(c) 0 (d) can not be found

85. The series $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$ is:

(a) conditionally convergent (b) absolutely convergent

(c) divergent (d) none of the above

86. $\int_0^1 (\sum \frac{x^n}{n^2}) dx =$

(a) $\sum_{n=0}^{\infty} \frac{1}{n(n+1)}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2(n-1)}$

(c) $\sum_{n=0}^{\infty} \frac{1}{n(n-1)}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n^2(n+1)}$

87. The only function that is analytic from the following is:

(a) $f(z) = sinz$ (b) $f(z) = \bar{z}$

(c) $f(z) = Im(z)$ (d) $R(\iota z)$

88. The mapping $\omega = z^2 - 2z - 3$ is:

(a) conformal within $|z| = 1$ (b) not conformal at $z = 1$

(c) not conformal at $z = -1$ and $z = 3$ (d) conformal everywhere

89. If $f(X) = X + 2/k$, $X = 1, 2, 3, 4, 5$ is the probability distribution of a discrete random variable, then k is:

(a) $-2/3$ (b) $5/8$

(c) $-5/7$ (d) $2/5$

90. If (X, d_1) and (Y, d_2) are two metric spaces and $f : X \to Y$ be a function such that the inverse image of closed set Y is closed in X, then:

(a) f is continuous function (b) f is closed function

(c) f is closed and continuous function (d) f is homeomorphism